Indian Statistical Institute, Bangalore
B. Math (II)
Second Semester 2010-11
Backpaper Examination : Statistics (II)

Date: 20-07-2011 **Maximum Score 70** **Duration: 3 Hours**

1. Incoming calls to the EPBX, managed by Ms. Merry, may be modelled by *Poisson process with parameter* λ, $\lambda > 0$. Ms. Merry is interested in estimating $\psi(\lambda)$, the probability of at most k calls in an interval of 10 units of time. The information would help Ms. Merry to decide on appropriate breaks from her work. Let $X_1, X_2, ..., X_n$ denote the number of incoming calls in *one* unit of time recorded on n different occasions.

   (a) State clearly the assumptions you make.

   (b) Can you assist Ms. Merry to obtain the required probability $\psi(\lambda)$?

   (c) Show that $T = \sum_{i=1}^{n} X_i$ is a minimal sufficient statistic for λ.

   (d) Is $T = \sum_{i=1}^{n} X_i$ complete as well? Substantiate.

   (e) Find Fisher information $I(\lambda)$ contained in the sample $X_1, X_2, ..., X_n$ about λ.

   (f) Find an unbiased estimator for $\psi(\lambda)$. Hence or otherwise obtain *UMVUE* for $\psi(\lambda)$.

   $[2 + 2 + 3 + 4 + 3 + 6 = 20]$

2. The manufacturer of a certain type of automobile claims that under typical urban driving conditions the automobile will travel at least 20 *km* per liter of petrol. The owner of an automobile of this type notes the mileages that she obtained in her own urban driving conditions when she fills the tank with petrol on 9 different occasions. She finds that the results *km per liter*, on different occasions were as follows :

   $15.6, 18.6, 18.3, 20.1, 21.5, 18.4, 19.1, 20.4, 19.0.$

   (a) List carefully the assumptions you must make and formulate the problem of testing of hypotheses to ascertain the manufacturer's claim.

   (b) Carry out a test at 5% level of significance.

   (c) Report the *p-value.*

   (d) Find 90% *confidence interval* for the expected distance travelled per liter of petrol.

   $[3 + 6 + 2 + 3 = 14]$

[PTO]

3. Let $X_1,\ X_2, ..., X_n$ be a random sample from $N(\theta,\ 1)$, the mean θ being unknown. Consider the problem of testing the hypotheses $H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$

   (a) Show that the family of densities of $\overline{X}$ possesses *monotone likelihood ratio (MLR) property.*

   (b) Show that the power function $\beta(\theta)$ of the test that rejects H_0 if $\overline{X} \leq c$ is decreasing in θ, where $c = \theta_0 - \frac{1}{\sqrt{n}} z_{(1-\alpha)}$ and z_p is the *p th quantile* of $N(0,\ 1)$ distribution.

   (c) Obtain the *size* of the above test in *(b)* and show that the test is *unbiased.*

   (d) **Derive** a level α *likelihood ratio test (LRT)* for testing the hypotheses $H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$.

   (e) Are the tests in *(b)* and *(d)* the same?

$[3 + 3 + 3 + 6 + 1 = 16]$

4. Let $X_n, Y_n, n \geq 1$ be sequences of random variables and X be a random variable, all defined on the same probability space, such that $X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{p} c$, where c is a finite constant. Prove that $X_n + Y_n \xrightarrow{d} X + c$.

[12]

5. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from *Bernoulli(θ)*, $0 < \theta < 1$. Let $\overline{X}_n = \frac{1}{n}\Sigma_{i=1}^{n} X_i$.

   (a) Obtain *maximum likelihood estimator for θ.*

   (b) Obtain $W_n = W_n(X_1,\ X_2, \cdots, X_n)$, *method of moments estimator, for $\tau(\theta) = Var(X_1)$.*

   (c) Does it agree with *maximum likelihood estimator for $\tau(\theta)$?*

   (d) *Asymptotic Normality(AN):* Show that W_n is $AN(\mu_n(\theta), \sigma_n^2(\theta))$ for suitable choice of $\mu_n(\theta)$ and $\sigma_n^2(\theta)$.

   (e) Show $n\sigma_n^2(\theta) \to \sigma^2(\theta)$ for some $\sigma^2(\theta) > 0$.

   (f) Find $\lim_{n\to\infty} \left[\sqrt{n}\left(\mu_n(\theta) - \tau(\theta)\right)\right]$.

   (g) Is your estimator W_n asymptotically unbiased?

   (h) Is your estimator W_n asymptotically efficient?

$[3 + 3 + 1 + 5 + 2 + 2 + 2 + 6 = 24]$